# An Approach for Contour Following 

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#### Abstract

In this paper，a new contour following approach is proposed to deal with the broken edge gaps in the binarized images．The new approach makes it possible to skip the gaps by means of taking adaptive steps．Hence the real contours can be found．In this approach，the boundaries are out put in forms of polygons which have the advantage that density of sample points is closely re lated to the curvat ure of boundaries．


Key words：Contour follow ing，Edge detection，Brokerr edge

## 0 Introduction

Boundaries of objects are perhaps the most important part in the hierarchy of struc tures that links raw image data with their ir terpretation．The solutions of many problems in computer vision such as object recognition， 3D reconstruction are based on the boundary detection．In the past years，many approaches have been proposed for detecting the bound aries of objects．How ever，for the complexity of images，each approach turns out to have its limitations．U sually one approach only suits one type of images ${ }^{[1]}$ ．Chain－coding is such a famous contour follow ing approach for finding continuous boundaries in binary images ${ }^{[2,4]}$ ．

Fig． 1 shows the path traced out by the chairr coding approach．If the boundaries of objects in gray level images（ we name them ob ject boundaries）are required to be traced，the gray level image is usually binarized first to produce their binary images so that they can be processed with chairr coding approach．Since the boundaries of inner region in the binary images（we might as well name them binary boundaries）are exactly traced if adopting the chair coding method，the quality of the bina
rization casts a critical effect upon the results of future work．The traditional approach of bri narization is to differentiate the background and the objects by choosing a threshold based on the histograms of the images．But the glob－ al thresholding approach may result in the loss of many partial features，and the spurious bri nary images would probably thus be pro－ duced ${ }^{[5]}$ ．To avoid this，sorts of local adaptive thresholding approaches have been taken， which work well to certain extent，but still have their limitations ${ }^{[6]}$ ．For example，they are not applicable to some complicated images． As in the case of the medicine image show $n$ in Fig．7，owning to the existence of broken edges，even the correct binarization under the optimal rules would produce spurious bound－ aries which are not our real need（as show n in Fig．2，the shaded part is the needed object， and the region encompassed by the dotted lines comes from the binarization）．For this type of binary images，the employment of chair cod ing approach would undoubtedly lead to a re－ sult that the boundaries detected are not of the object we need，but the binary boundaries，be－ cause this approach is based on the theory of boundary continuity（i．e．the dotted line in Fig．2）．To overvome the aforementioned limi

[^0]tations, we propose a new approach for corr tour following suitable for the brokerr edged images in particular.


Fig. 1 The path traced out by the chair coding approach

broken edge
Fig. 2 The shaded part is the needed object, ant the region encompassed by the doted lines comes from binarization

## 1 A New Approach for Contour Following

In this approach, a closed polygon $P_{0} P_{1} \ldots P_{n}$ is used to approximate a boundary $\Gamma$ of a given region in a binary image. Suppose $\Gamma$ is a simple closed curve $F(t)$ counterclockw ise ordered, when a point $P_{i}=$ $F\left(t_{i}\right)$ is found, the next point $P_{i+1}=F\left(t_{i}+\right.$ $\Delta t$ ) can be expressed as Taylar formula:
$F\left(t_{i}+\Delta t\right)=F\left(t_{i}\right)+F^{\prime}\left(t_{i}\right) \Delta t+$ $\frac{F^{\prime \prime}\left(t_{i}\right)}{2} \Delta t^{2}+o\left(\Delta t^{3}\right)$

So, to find the next point $P_{i+1}$, we first move from $P_{i}$ to a new point $Q_{i+1}^{0}=F\left(t_{i}\right)+$
$F^{\prime}\left(t_{i}\right) \Delta t$ by taking one step at the length $\delta$ along the tangent $T_{i}^{0}=F^{\prime}\left(t_{i}\right)$, then we try to search a series of points $Q_{i+1}^{1}, Q_{i+1}^{2}, \cdots$, $Q_{i+1}^{l}$ for $F\left(t_{i}+\Delta t\right)$ in the direction of $F^{\prime \prime}\left(t_{i}\right)$ : If $Q_{i+1}^{0}$ is in the outer region( inner region), we let $Q_{i+1}^{j} 1_{i+1}^{j+1}$ be perpendicular to $P_{i} Q_{i+1}^{j}$ countercl $\sigma$ ckwise( clockwise) and the length of $Q_{i+1}^{j} Q_{i+1}^{i+1}$ be $\eta$ until $Q_{i+1}^{l}$ is a point in the inner region( outer region) ( $j$ starts from 0), finally, we take $Q_{i+1}^{l-1}$ as $P_{i+1}$. In the above procedure, the initial point $P_{0}$ and its tangent $T_{0}^{0}$ are given. Since it is difficult to find $T_{i}^{0}$, we always take $P_{i} P_{i+1}$ as $T_{i}^{0}$. The formula of the approach is thus as follows.

Suppose $\Gamma$ is the binary boundary, and $P_{0}$ is a point on the boundary, $T_{0}^{0}$ is tangent vector of $\Gamma$ at $P_{0}(\Gamma$ is assumed to be counterclockwise). Let tangent step be $\delta$, normal step be $\eta$, then the procedure of constructing polygon $\widetilde{\Gamma}$ to approximate $\Gamma$ is shown in Fig. 3.

Starting with $P_{0}$, let $Q_{0}^{0}=P_{0}+\delta T_{0}^{0}$, we might assume that $Q_{0}^{0}$ is in the outer region (inner region). Let $N_{0}^{0}$ be the normal vector of $T_{0}^{0}, Q_{0}^{1}=Q_{0}^{0}+\eta \mathrm{N}_{0}^{0}$, if $Q_{0}^{1}$ is still in the outer region(inner region). Let $T_{0}^{1}=$ $\frac{Q_{0}^{1}-P_{0}}{\left\|Q_{0}^{1}-P_{0}\right\|}, N_{0}^{1}$ the normal of $T_{0}^{1}$, then $Q_{0}^{2}$ $=Q_{0}^{1}+\eta N_{0}^{1}$, if $Q_{0}^{2}$ is also within the outer region (inner region). Let $T_{0}^{2}=$ $\frac{Q_{0}^{2}-P_{0}}{\left\|q_{0}^{2}-P_{0}\right\|}, N_{0}^{2}$ the normal of $T_{0}^{2}, Q_{0}^{3}=Q_{0}^{2}$ $+\eta N_{0}^{2}$, we can go on like this until $Q_{0}^{i}$ comes into inner region (outer region). Let $P_{1}=$ $Q_{0}^{i-1}$, the approximate tangent vector $T_{1}^{0}=$ $\frac{Q_{1}-P_{0}}{\left\|P_{1}-P_{0}\right\|}$, then starting with $P_{1}$, we get point $P_{2}, P_{3}, \cdots$ Repeat this procedure, until the distance betw een $P_{n}$ and the starting point $P_{0}$ is smaller than a fixed value. We can thus get an approximate polygon $\widetilde{\Gamma}=$ $P_{0} P_{1} P_{2} \ldots P_{n} P_{0}$ of the boundary $\Gamma$. The normal vector $N^{i}$ can be obtained in the following way:
$\left\{\begin{array}{l}N_{i}^{j}=\left(\begin{array}{rr}0 & -1 \\ 1 & 0\end{array}\right] T_{i}^{j} \text {, if } Q_{i} \text { is in the outer region } \\ N_{i}^{j}=\left(\begin{array}{cr}0 & 1 \\ -1 & 0\end{array}\right] T_{i}^{j} \text {, if } Q_{i} \text { is in the inner region }\end{array}\right.$ But problems also arise in this procedure:
(1) When the tangent step is larger than the diameter of inner region, it is impossible to carry out the contour following procedure, as shown in Fig. 4.
(2) How to ensure the error between $\widetilde{\Gamma}$ and $\Gamma$ with the precision requirement.


Fig. 3 The procedure of the new approach


Fig. 4 The situation of tangent step being too large

To solve these problems, some relation ships should be established between tangent step and the boundary curvature. As the curvature calculation is time consuming, we deal with it by menas of taking adaptive tangent step. As shown in Fig. 5, starting with $P_{i}$, the contour is traced at the tangent step $\delta_{0}$, normal step $\eta$, until $Q_{i}^{2}$ is reached. If $Q_{i}^{2}$ is
still away from the inner region, we just reduce the tangent step by half while the normal vector remains the same. Restart from point $P_{i}$, and repeat the procedure unt il the point $Q_{i}^{2}$ comes into the inner region, then, let $P_{i+1}=Q_{i}^{1}$ and recover the tangent step to be $\delta_{0}$. In this way, through the adaptive tangent step, the contour following procedure will go on when there is no broken edges on the boundaries. Meanwhile, it is possible to control the error to be smaller than the normal step.

When there are broken edges on the boundary of object, the region produced by br narization would be probable not that of the real object. If so, a narrow gap perpendicular to the broken edge would appear. It is the extension of the inner region formed by binariza tion. So, we call it broken edge gap ( as shown in Fig. 2). The follow ing part of this paper is about how to detect the contour near the broken edge gap by means of taking adaptive steps.


Fig. 5 The procedure of taking adapt ive tangent step

When the boundary detection comes to point $P_{i}$ from which the distance to the broken edge gap is smaller than the original tangent step $\delta_{0}$, three cases are likely to take place.

In case one, $Q_{i+1}^{0}$ comes to the other side of the broken edge gap and $Q_{i+1}^{2}$ obtained by the above procedure comes to the inner region, so $P_{i+1}$ has traversed the gap at the original tangent step $\delta_{0}$, and no special treatment is
needed.


Fig. 6 The case two of encountering broken edge gap

In case two,$Q_{i+1}^{0}$ comes to the other side of the broken edge gap, but $Q_{i+1}^{2}$ has not come to the inner region. According to the above procedure, we must take adaptive step to get new $Q_{i+1}^{0}$. So $P_{i+1}$ obtained is pulled back to the same side of the broken edge gap where $P_{i}$ is located, and it is closer to the gap, as shown in Fig. 6.

In case three, $Q_{i+1}^{0}$ is in the inner region, which indicates that $Q_{i+1}^{0}$ has come into the broken edge gap. The $P_{i+1}$ obtained is similar to that in the case two.

In the latter two cases, the following situations are possibly to arise:

Continue the proceure to get $P_{i+2}, P_{i+3}$, ..., but no point is able to traverse the gap. The real tangent steps turn shorter and shorter when the procedure goes well. If no measures are taken, the detection will go along the binary boundary just as it will if we take the chair coding approach. Therefore, it is necessary to set a threshold vale $\delta_{\text {min }}$ for the tangent step. When the detecting step $\delta$ becomes smaller than $\delta_{\text {min }}$, we might consider that the broken edge gap has been reached, which calls for the special treatment. Suppose the detection has come to $P_{k}$, and judging from the tangent step, $P_{k}$ has reached the gap, then the follwing procedure is necessary for the purpose of making $P_{k+1}$ traverse the
gap.
Firstly, recover the current step $\delta$ to be the original tangent step $\delta_{0}$, let $Q_{k}^{0}=P_{k}+$ $\delta T_{k}^{0}$.

If $Q_{k}^{0}$ is proved in the outer region, then $Q_{k}^{0}$ can be regarded having traversed the gap. And we do not take adaptive step any more but let $Q_{k}^{i}=Q_{k}^{i-1}+\eta N_{k}^{0}$ and continue the detection along the normal vector unt il $Q_{k}^{i}$ goes into the inner region, then let $P_{k+1}=Q_{k}^{i-1}$.

If $Q_{k}^{0}$ is in the inner region, then $Q_{k}^{0}$ is shown having come into the broken edge gap, we could double the current step $\delta$, and give a new value to $Q_{k}^{0}=P_{k}+\delta T_{k}^{0}$ until the new $Q_{k}^{0}$ is in the outer region.

Hence, after selecting the suitable values for $\delta_{0}, \eta, \delta_{\text {min }}$ respectively according to different requirements, this approach can be applied to trace the contours with broken edges.

The complete algorithm of this approach can be summarized as follows:
step 1. Binarize the gray level image.
step 2. Select the original tangent step $\delta_{0}$, normal step $\eta$, and the gap judgment step $\delta_{\text {min }}$. Scan the image to obtain the first point on the boundary $P_{0}$ and figure out its approximate tangent vector $T_{0}^{0}$, let $i=0, \delta$ $=\delta_{0}$.
step 3. Let $Q_{i}^{0}=P_{i}+\delta T_{i}^{0}, N_{i}^{0}$ be the normal vector of $T_{i}^{0}$, let $Q_{i}^{1}=Q_{i}^{0}+\eta N_{i}^{0}$. Suppose $Q_{i}^{0}$ is in the outer region. If $Q_{i}^{1}$ enters the inner region, let $P_{i+1}=Q \frac{1}{i}$, turn to step 6; else, let $T_{i}^{1}=\frac{Q_{i}^{1}-P_{0}}{\left\|Q_{i}^{1}-P_{i}\right\|}, N_{i}^{1}$ be the normal vector of $T_{i}^{1}$. Let $Q_{i}^{2}=Q_{i}^{1}+\eta N_{i}^{1}$, if $Q_{i}^{2}$ enters the inner region, let $P_{i+1}=Q_{i}^{2}$, turn to step 6; else turn to step 4.
step 4. To see if the broken edge gap is reached. If $\delta<\delta_{\min }$, turn to step 5 for broken edge processing; else, half $\delta$ and turn to step 3.
step 5. Let $\delta=\delta_{0}, Q_{i}^{0}=P_{i}+\delta T_{i}^{0}$, if $Q_{i}^{0}$ is in the inner region, double $\delta$ and let $Q_{i}^{0}$
$=P_{i}+\delta T_{i}^{0}$ until $Q_{i}^{0}$ enters the outer region; else let $Q_{i}^{j+1}=Q_{i}^{i}+\mathrm{V}_{i}^{j}($ start $j$ from 0$)$, until $Q_{i}^{i+1}$ enters the inner region, let $P_{i+1}=$ $Q_{i}^{j}$, go to step 6.
step 6. If $\left\|P_{i+1}-P_{0}\right\|<\delta_{0}$ and $i>$ $\delta_{0} / \delta_{\text {min }}$, terminate and output the polygon $P_{0} P_{1} P_{2} \ldots P_{i+1} P_{0} ;$ else, let $i=i+1, \quad \delta=$ $\delta_{0}$, turn to step 3.

## 2 Discusssion and Conclusion

As mentioned above, the approach pro posed makes it possible to automativally detect the contour of the image with broken edges by traversing the broken- edge gap generated from the binarization of the brokerr edged image. Apart from that, it also has some other advarr tages in comparison with the chairr coding ap proach.

The first advantage lies in the boundary representation. Output of chairr coding algo rithm is the boundary in the form of chain codes. The future work such as 3D reconstruc tion needs the polygon representation of the object contour. Although chain code can also be treated as a special form of the polygon rep resentation, there is self intersection result ing from the algorithm, as shown in Fig. 1. Even if there is no self intersection in the object boundary itself, the chain codes would get self intersected sometimes. Besides, the appearance of coincident edges is another possible sit uat ion. So chain codes can not be applied di rectly as a polygon, and the polygonal approxi mation must be performed for its future use. Whereas the output of the proposed approach directly employs the polygon representation of the boundary and adapts the adaptive steps which make sample points denser on the boundary where the curvature is larger, and sparser where it is smaller. In such a good surrounding, the 3 D reconstruction will be ut $\dot{r}$ lized effectively, which is difficult to achieve for the polygon approximation of chain codes ${ }^{[3]}$. In addition, the boundary description of objects should sometimes be converted into
area description in image processing. Because of the self intersection, some special treatments should be given to the chain codes representation ${ }^{[8]}$. But the output of the new approach can be directly applied to the scanning conversion.

The second advantage comes from the computation speed. This algorithm enables the detection to go on at the much larger searching step than that in chairr coding, so the time consumed focuses on the normalization of the tangent vector and normal vector, which might be a little bit more than that cost in chain coding. However, if the outputs in two algorithms are both required to be the polygons, it will be much quicker in the proposed algorithm than in polygon approximation.

We just apply the approach to the procession of medicine images and make a comparison with the chairr coding method. Fig. 7 shows the result of chairr coding detection, in which mist akes arise when it


Fig. 7 Contour traced by chairr coding approach Mistakes arise at the broken edge
comes to the broken edge. Fig. 8 shows that the brokerr edge gap can be traversed if adopt ${ }^{-}$ ing the new approach, and the boundary we need can thus be obtained.

This approach is mainly for brokerr edged image processing. When a narrow area does exist in the image object and the exact bound ary of the binary image must be detected, we can simply fix the tangent step and the normal
step to be 1 , and then use this new method to obtain the required results.


Fig. 8 Contour traced by the new approach Broken edge gap can be traversed

## References

[1] Ballard D H, Brown C M. Computer Vision, PrenticeH all. Inc. 1982
[2] Baruch O. Senmentation of 2-D Boundaries using the chain code, V isual Communications and $\mathrm{Im}^{-}$ age Processing, 159
[3] Duda R O, Hart P E. Commun. ACM, 1972, 15( 1): 11
[4] Duda R O, Hart P E. Pattern Recog nition and Science Analysis. New York: Wiley 1973
[5] Glasbey C A. Graphical Models and Image Processing, 1993, 55(4): 532
[6] Wilkinson M H F. Graphical Models and Image Processing, 1998, 60(6): 385
[7] Ren Z, Ameling W, Jensch P. An Attributed $T$ reedata structure for Representing the Descriptions of Object Contours in Image, V isual Communications and Image Processing' 90, Part2, 956
[8] Shih F Y, Wong W T. Graphical Models and Im age Processing, 1994, 56(4): 348

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